where

$$\mu = 1 - \frac{\lambda B^2}{2\beta R}$$

$$\eta = \frac{2B^2K_0^2}{\zeta R}(1+\alpha)\left(\frac{K_0}{2} - \frac{\lambda_2}{2\lambda_3}\right)$$

$$\zeta = (4\lambda_1\lambda_3 - \lambda_2^2)^{1/2} \tag{21}$$

and A_0 is a constant of integration Equations (1) and (6) give ρ_p, ρ_g, and P_g

It may be concluded that, although this method does not give the exact solution to a specific problem directly, it is very useful in discussing the influence of the various parameters on performance

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Area Change with a Free-Piston Shock Tube

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THE free piston shock tube has been shown[1 2] to be a viable alternative to the arc-driven shock tube as a means of achieving shock Mach numbers substantially in excess of 20 in air It differs from the conventional shock tube in that the driver gas is initially contained in a large "compression tube" and, immediately prior to rupture of the shock-tube diaphragm, is heated and raised to high pressure through compression into a driver section of much smaller volume by motion of the free piston along this tube

To date, this modification has been considered only in conjunction with a constant-area shock tube The length of the shock-tube driver section should be at least a few times greater than the diameter to insure that the flow is not grossly distorted by the diaphragm opening process, and, therefore, when high volumetric compression ratios are desired in the driver gas, the diameter of the shock tube must be much smaller than the diameter of the compression tube Because reduction in diameter has severe adverse effects on shock-tube flow, this constitutes an important limitation on free-piston shock tubes In this note, it is shown that this limitation can, in principle, be removed by increasing the cross-sectional area in passing from the driver to the driven section, a technique initially employed by Lin and Fyfe[3] in a shock tube with combustion driver

The piston is sufficiently massive that it remains effectively stationary for the duration of the shock-tube flow, and so a wave diagram can be drawn, as in Fig 1, by following Lin and Fyfe In the usual notation, with $\gamma_4 = \gamma$, the driver-interface pressure ratio can, therefore, be written as

$$\frac{P_3}{P_4} = \left[f\left(1 - \frac{\gamma-1}{2f}\frac{u_3}{a_4}\right)\right]^{2\gamma/(\gamma-1)} \tag{1}$$

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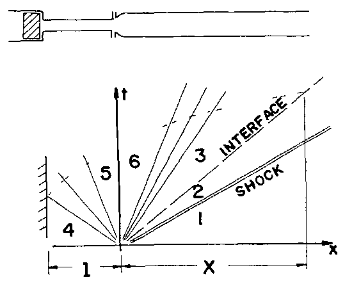

Fig 1 Wave diagram for shock-tube flow

where

$$f = \left(\frac{2}{\gamma+1}\right)^{1/2}\left(1 + \frac{\gamma-1}{2}M_6\right)\left(1 + \frac{\gamma-1}{2}M_6^2\right)^{-1/2}$$

and

$$\frac{A_1}{A_4} = \frac{1}{M_6}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M_6^2\right)\right]^{(\gamma+1)/2(\gamma-1)}$$

M_6 is the flow Mach number at the downstream end of the area change From Eq (1), an expression may be obtained comparing driver-gas volumetric compression ratios in constant-area and change-of-area shock tubes, when both produce the same interface velocity (and hence the same shock Mach number) with the same pressure ratio P_4/P_3, i e ,

$$(R'/R)^{(\gamma-1)/2} = a_4'/a_4 = 1 - (1-f)[1 - (P_3/P_4)^{(\gamma-1)/2\gamma}]^{-1}$$

where primed symbols indicate quantities associated with the constant-area shock tube, and R is the driver-gas volumetric compression ratio This is used to plot R/R' and l/l', where l is the length of the driver section, in Figs 2a and 2b, the curves being terminated when the available pressure ratio becomes insufficient to produce fully expanded flow through the area change

The limited volume allowed for the driver section implies that special attention must be paid to the distance X along the shock tube at which useful flow is terminated by the head of the expansion wave reflected from the upstream end of the driver overtaking the interface Calculation of the (x, t) equation for this "reflected head" is a straightforward procedure (e g , Ref 4), and, when it is combined with the (x, t) equation for the interface, it is found that

$$\frac{X}{l} = \frac{u_3}{a_4}(1+M_6)\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M_6\right) \times \left(1 - \frac{\gamma-1}{2f}\frac{u_3}{a_4}\right)\right]^{-(\gamma+1)/2(\gamma-1)}$$

or, comparing for identical performance as in the foregoing,

$$\frac{X}{X'} = \left(\frac{a_4'}{a_4}\right)^{(\gamma+1)/(\gamma-1)}\frac{1+M_6}{2M_6} \times \left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M_6^2\right)\right]^{(\gamma+1)/4(\gamma-1)}$$

This ratio is plotted in Fig 2c

The curves show that, where a choice is possible, a constant-area shock tube is to be preferred to a change of area, since the latter requires a higher volumetric compression ratio to achieve the same performance and roughly the same duration of useful flow It is when constant area leads to a driver section that is too short that area change becomes important, enabling performances that could not otherwise be achieved For example, using a compression tube 10 ft long and 3 in in diameter with helium driver gas, Ref 2 indicates that a shock Mach number of 30 in air can be achieved, with $P_4/P_3 = 10^3$, in a 2-in -diam constant-area

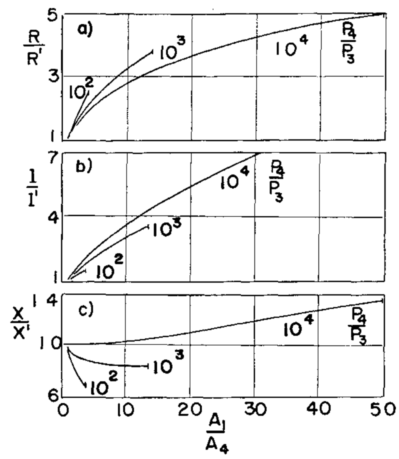

Fig 2 Compaiison of area change with constant-area shock tube, γ = 1 67

shock tube only if the driver is 2 5 in long From Fig 2, a 1-in -diam driver could be 4 5 in long

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Entropy Production in Vibrational-Nonequilibrium Nozzle Flow

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IT is well known that the existence of nonequilibrium in a flow system serves as a source of internal entropy generation This note is concerned with the entropy production resulting from vibrational nonequilibrium in a hypersonic nozzle and is based on the numerical calculations presented in Ref 1

That work presented a quasi-one-dimensional analysis of the vibrational-nonequilibrium flow of nitrogen in a hypersonic nozzle having a geometry given by

$$A/A^* = 1 + (\tan\theta/r^*)^2 x^2 \tag{1}$$

where A is the nozzle cross sectional area, A^* the throat area, θ the nozzle half-angle, r^* the nozzle throat radius, and x the distance from the throat along the nozzle axis

Equilibrium was assumed to exist from the stagnation chamber to the nozzle throat, and the vibrational relaxation during the remainder of the expansion was governed by the rate equation

$$d\sigma/dt = (\sigma \ - \sigma)/\tau \tag{2}$$

where σ is the vibrational energy, the subscript e denotes equilibrium, and the vibrational-relaxation time for nitrogen, τ, is given by

$$\tau p = AT^{1/2}\exp(B/T^{1/3}) \tag{2a}$$

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where the constants A and B are those obtained in Ref 2 by an empirical correlation of data

The degree to which the flow parameters in the nozzle are affected by the existence of nonequilibrium was shown In order to show these effects as a function of initial stagnation pressure p_0 and nozzle size and geometry, the correlating group $L = p_0(r^*/\tan\theta)$ in units of atmosphere-centimeters was varied in that investigation (for a representative case where $r^* = 0\ 125$ cm, $\theta = 10°$ and $p_0 = 1000$ atm, $L = 709$)

In a review of Ref 1, Presley[3] suggested that the work be extended to include the calculation of the entropy production in the nozzle-flow process This note deals with such a calculation In the present study, the following model is used to describe the gas system in vibrational nonequilibrium The energy modes are divided into two subsystems The first of these contains only the translational and rotational energy modes Thus the temperature of this subsystem is the translational temperature T The second subsystem represents the vibrational energy and is assumed to be composed of harmonic oscillators with a Boltzmann distribution with respect to energy levels Thus a vibrational temperature T can be defined by the relation

$$\sigma = \frac{R\theta_v}{\exp(\theta\ /T\ ) - 1} \tag{3}$$

where θ is the characteristic temperature of molecular vibration and R is a gas constant By considering the energy exchange between the two subsystems, it can be shown[4] that the internal entropy generation due to vibrational nonequilibrium is given by

$$d_i s = [(1/T\ ) - (1/T)]d\sigma \tag{4}$$

Now consider Eq (4) with relation to inviscid flow in a hypersonic nozzle Throughout this expansion process, $d\sigma \leq 0$ For the two limiting cases of equilibrium flow and frozen flow, the expansion will be isentropic, since for the equilibrium case $T\ = T$ and for the frozen case $d\sigma = 0$

For the intermediate case considered in Ref 1, the flow from the stagnation chamber to the throat was assumed to be in equilibrium, that is, $T = T$, so that in this initial phase of the expansion $d_i s = 0$ Then, as further expansion occurs beyond the throat, vibrational-nonequilibrium effects can come into play These occur when rate of vibrational adjustment is too slow to allow the vibrational temperature T to decrease as rapidly as the translational temperature T It can be seen from Eq (4) that this must result in an increase in entropy As the expansion proceeds, the pressure and temperature of the gas continually decrease, resulting in a corresponding increase in relaxation time τ Consequently, $d\sigma/dt$ decreases and eventually becomes negligible, so that further expansion occurs isentropically

To determine the entropy increase from the stagnation chamber to any position l in the nozzle, Eq (4) can be integrated numerically from $x = 0$ to $x = l$:

$$\Delta\frac{s_i}{R} = \frac{1}{R}\int_0^l\left(\frac{1}{T\ } - \frac{1}{T}\right)\frac{d\sigma}{dx}\,dx \tag{5}$$

Using Eq (5), the entropy production in vibrational-nonequilibrium flow of nitrogen was evaluated for a single stagnation condition, $p_0 = 100$ atm, $T_0 = 4000°$K, and a range of values of L The values of T, T , and $d\sigma/dx$ vs x needed to carry out the integration were obtained from the numerical computations that were made for Ref 1 The results obtained by numerical integration are shown in Fig 1 The line of constant stagnation enthalpy in the upper portion of the figure is based on equilibrium relationships This is consistent with the assumption of equilibrium in the stagnation chamber and with the definition of a total pressure The remainder of the plot simply shows the entropy vari-